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ROBUST STABILITY OF SYSTEMS WITH INTEGRAL CONTROL

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Abstract

A necessary and sufficient condition is derived which must be satisfied by the plant steady state gain matrix of a linear time invariant system in order for an integral controller to exist for which the closed loop system is unconditionally stable. Based on this theorem the robustness of integral control systems is analyzed, i.e. the family of plants is defined which are stable when controlled with the same integral controller. Conditions for actuator/sensor failure tolerance of systems with integral control are also given. Finally, parallels are drawn between the results of this paper and the bifurcation theory of nonlinear systems.

Introduction

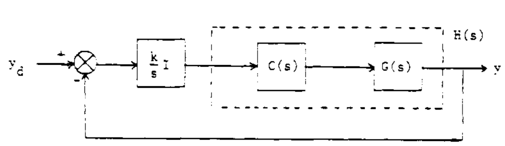
Process control, and in particular chemical process control, is characterized by open-loop stable and sluggish processes, severe modelling problems and the overriding need for reliability, robustness and good steady state performance of the control system, i.e. negligible offset. In order to reduce the system sensitivity at $\omega = 0$ to a small value, controllers with integral action are typically employed in all important situations. Therefore the modelling requirements for the design of controllers with integral action, their robustness in the event of plant changes and their tolerance to actuator and/or sensor failure are of significant practical interest.

Unless stated otherwise we will assume throughout the paper that the plant is an open loop stable, linear time invariant system. Let G(s) denote the plant transfer matrix. We will assume that the plant is functionally controllable [1], i.e. that the right inverse of G(s) exists, because only then it is possible to install controllers with integral action on all the outputs. For simplicity in notation but without loss of generality we will restrict G(s) to be a square matrix relating n inputs to n outputs.

We will use the following notation: $\underline{C}^+$ is the open right half complex plane; A^{ij} is the matrix A with the ith row and the jth column removed, $\lambda_j(A)$ and det(A) are the jth eigenvalue and the determinant of the matrix A respectively.

Integral Controllability

The basic control system configuration is shown in Fig. 1



Here G(s) and C(s) are the transfer matrices of the plant and the dynamic compensator respectively, both of which are assumed to be strictly stable. I is the identity matrix and k is a positive constant. We define H(s) = G(s)C(s) and H'(s) = C(s)G(s). In this paper we would like to address the following questions: What are the requirements on H(s) or equivalently, how does the compensator C(s) have to be designed, for a positive k to exist for which the closed loop system is stable? How tolerant is a control system of this type to plant changes and actuator and/or sensor failures? For this purpose we will introduce the following definition.

Definition 1: The open-loop stable system H(s) is called integral controllable if there exists a $k^* > 0$ such that the closed loop system shown in Fig. 1 is stable for all k satisfying $0 < k \le k^*$.

It is important to note that we exclude conditionally stable systems in this definition. There could be a $k = k' > 0$ for which the system in Fig. 1 is stable. But unless k' can be made arbitrarily small, the system is not integral controllable according to our definition. Conditionally stable systems which are only stable for high gains k are undesirable from a practical point of view. We will discuss this issue in more detail later.

The following theorem is the main result of this paper and forms the basis of all subsequent theorems on robustness and failure tolerance.

Theorem 1: The system H(s) is integral controllable if and only if all the eigenvalues of H(0) lie in the open right half complex plane.

Proof: Let the Nyquist D-contour be indented at the origin to the right to exclude the pole of 1/s H(s) at the origin. The system will be closed loop stable if none of the characteristic loci (CL) [2] encircles the point (-1/k,0). For integral controllability it is necessary and sufficient that the CL intersect the negative real axis only at finite values. An intersection at $(-\infty,0)$ could only occur because of the pole of 1/s H(s) at the origin. Along the indentation, the small semi-circle with radius ε around the origin, the CL can be described by

$$\lambda_j(H(0)) \cdot \frac{1}{\varepsilon} e^{i\phi} \qquad -\frac{\pi}{2} \le \phi \le \frac{\pi}{2}; \qquad j = 1,n \tag{1}$$

for small ε. Let $\lambda_j(H(0)) = r_j e^{i\theta_j}$ then the expression can be rewritten as

$$\frac{r_j}{\varepsilon} e^{i(\theta_j+\phi)} \qquad -\frac{\pi}{2} \le \phi \le \frac{\pi}{2} \qquad j = 1,n \tag{2}$$

The CL do not cross the negative real axis if $-\pi < \theta_j + \phi < \pi$ or $-\frac{\pi}{2} < \theta_j < \frac{\pi}{2}$ which means $\lambda_j(H(0)) \in \underline{C}^+$, $j = 1,n$.

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Corollary 1: H(s) is integral controllable only if $\det(H(0)) > 0$.

It would be useful to know how to design the compensator C(s) such that the system is integral controllable. A possibility is to choose C(s) such that $C(0) = G(0)^{-1}$, that is, to completely "decouple" the system at the steady state. In practice we often like to reduce the complexity of C(s) and to restrict its structure, for example to the following form: C(0) = PD, where D is a diagonal matrix of constants and P a permutation matrix. This form of C(0) would imply that a set of single-input-single-output controllers can be used. Unfortunately it is not clear if a compensator of this structure which makes the system integral controllable exists for all plants G(s) or what conditions G(0) must satisfy for such a C(0) to exist. A related but somewhat more restrictive result is derived in the next section.

Failure Tolerance

Obviously for any actuator or sensor failure the system shown in Fig. 1 will be unstable because of the integral control action. However, the following definitions with regard to Fig. 2 A and B allow a meaningful problem formulation (K is a diagonal matrix, $K = \mathrm{diag}(k_1, \ldots k_n)$).

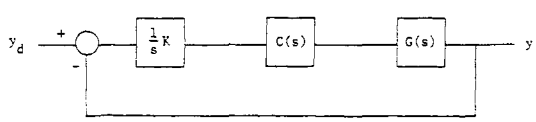

Fig. 2A

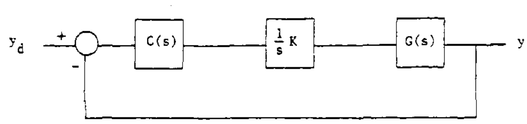

Fig. 2B

Definition 2: The system shown in Fig. 2A is j-sensor failure tolerant (j-SFT) if it is integral controllable and if there exists a $k^* > 0$ such that the closed loop system is stable for all $0 < k \leq k^*$; $k_i = k$; $i = 1,n$; $i \neq j$; $k_j = 0$.

Definition 3: The system shown in Fig. 2B is j-actuator failure tolerant (j-AFT) if it is integral controllable and if there exists a $k^* > 0$ such that the closed loop system is stable for all $0 < k \leq k^*$; $k_i = k$; $i = 1,n$; $i \neq j$; $k_j = 0$.

In these failure tolerance definitions we assume that the failure has been recognized and that the loop with the faulty sensor or actuator has been taken out of service, i.e. the specific integrator has been removed and k_j has been set to zero. We require that without readjustment of the other part of the control system, system stability be preserved.

Theorem 2: The system shown in Fig. 2A(B) is j-SFT (j-AFT) if and only if $\lambda_i(H(0)) \varepsilon \underline{C}^+$, $i = 1,n$ and $\lambda_i(H(0)^{jj}) \varepsilon \underline{C}^+$, $i = 1, n-1$ ($\lambda_i(H'(0)) \varepsilon \underline{C}^+$, $i = 1,n$ and $\lambda_i(H'(0)^{jj} \varepsilon \underline{C}^+$, $i = 1,n-1$).

Proof: Theorem 2 follows directly from Theorem 1.

Corollary 2: The system shown in Fig. 2A(B) is j-SFT (j-AFT) only if $\det(H(0)) > 0$ and $\det(H(0)^{jj}) > 0$ ($\det(H'(0)) > 0$ and $\det(H'(0)^{jj}) > 0$.

Obviously, analogous conditions can be stated for the case of several simultaneous sensor or actuator failures. Again the choice $C(0) = G(0)^{-1}$ would provide tolerance to arbitrary failures. The question of a simplified structure for C(0) is of interest. Here we can state a necessary condition to be satisfied by G(0) for a diagonal compensator C(0) to exist.

Let the elements of G(0) be denoted by g_{ij} and the elements of $G(0)^{-1}$ by $\hat{g}_{ij}$. Define the matrix M with the elements

$$m_{ij} = g_{ij}\,\hat{g}_{ji} \tag{3}$$

M is called the Relative Gain Array (RGA) [3] and enjoys widespread use in process control as an interaction measure despite its empirical derivation. M can be easily shown to be invariant under input and output scaling of G and to satisfy

$$\sum_{i=1}^{n} m_{ij} = 1 \qquad j - 1,n$$
$$\sum_{j=1}^{n} m_{ij} = 1 \qquad i = 1,n \tag{4}$$

Theorem 3: A diagonal compensator C exists such that H(s) is 1) integral controllable; 2) j-AFT and j-SFT; 3) 1,2,...j-1, j+1,...n-AFT and 1,2,...j-1,j+1,...n-SFT; only if $m_{jj} > 0$.

Proof: Because m_{ij} is invariant under input and output scaling we have for any diagonal pre- or post-compensator C(0)

$$m_{ij} = (-1)^{i+1} g_{ij} \frac{\det(G(0)^{ji})}{\det(G(0))}$$
$$= (-1)^{i+j} h_{ij} \frac{\det(H(0)^{ji})}{\det(H(0))} \tag{5}$$

For property 1) $\det(H(0)) > 0$ (c.f. Cor. 1); for property 2) $\det(H(0)^{jj}) > 0$ (c.f. Cor. 2); for property 3) $h_{jj} > 0$ (c.f. Cor. 2); therefore for properties 1), 2) and 3) it is necessary that $m_{jj} > 0$. QED.

The condition $m_{jj} > 0$ is obviously not sufficient for properties 1) - 3) except when n = 2.

Corollary 3: A diagonal compensator C exists for the 2x2 system H(s) such that it is 1) integral controllable; 2) 1-AFT and 1-SFT; 3) 2-AFT and 2-SFT if and only if $m_{11} > 0$. Moreover, any 2x2 system can always be brought into a form such that $m_{11} > 0$ by a permutation of the inputs.

Proof: The necessity follows from Theorem 3. The sufficiency can be proved as follows

$$m_{11} = \frac{h_{11}(0)h_{22}(0)}{\det(H(0))} \tag{6}$$

There always exists a diagonal compensator C such that $h_{11}(0) > 0$ (property 3) and $h_{22}(0) > 0$ (property 2). Therefore $m_{22} > 0$ implies $\det(H(0)) > 0$. The eigenvalues of H(0) are the roots of

$$\lambda^2 - (h_{11}(0)+h_{22}(0))\lambda + \det(H(0)) = 0 \tag{7}$$

For this second order polynomial $\det(H(0)) > 0$ and $h_{11}(0) + h_{22}(0) > 0$ implies that all the eigenvalues of H(0) are in the RHP (property 1). Moreover, when $m_{11} > 0$ define

$$G' = G \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{8}$$

that is, exchange the system inputs. Then

$$m_{11}(G') = m_{12}(G) = 1 - m_{11}(G) > 0 \qquad \underline{\text{QED}} \tag{9}$$

It is very desirable to have a system which satisfies properties 1) - 3) because the task of designing the controller is greatly simplified. They imply that loop j is relatively independent of the remaining part of the system: The overall system can only be stable if loop j and the remaining system are stable by themselves. Roughly speaking, loop j and the other loops can be designed separately without paying much attention to the interactions. A negative diagonal element of the RGA ($m_{jj} < 0$) implies that loop j can only be designed with proper consideration given to all the other loops, i.e. that the system has severe interactions. Therefore in practice loop pairings which lead to negative m_{jj}'s are usually avoided. Unfortunately, except in the case of 2x2 system, positive m_{jj}'s do not imply that there are no interaction problems.

Robustness

The model of a plant is never perfect and therefore it is important that the control system is not only stable for the nominal plant but also for a family of plants in some "neighborhood" of the nominal plant. We would like to investigate the robust stability of plants with integral control. This property is enjoyed by a family of plants $\mathscr{P}$ if there exists a single compensator C which makes all the members of the family integral controllable. Let the transfer matrix of any member of the family be denoted by G(s) and the transfer matrix of the nominal plant by $G_0(s)$. We may define for each plant in the family the matrix function

$$\Pi(s) = G(s)G_0^{-1}(s) \tag{10}$$

The function $\Pi(s)$ can be interpreted as a multiplicative perturbation of the nominal plant. Then we have the following result:

Theorem 4: Suppose that the family of plants satisfies the following assumptions:

a) Each plant in $\mathscr{P}$ is open-loop stable
b) There exists a fixed square matrix N such that for each plant the matrix $\Pi(0)N$ has all its eigenvalues in a bounded region in the open right half complex plane

Then there exists a single compensator C and a single $k > 0$ such that each plant in $\mathscr{P}$ is stable with the control figuration shown in Fig. 1.

Proof: Theorem 4 is an immediate consequence of Theorem 1 when $C = G_0(0)^{-1}N$. Note that when the nominal plant $G_0(s)$ is also a member of the family, as is usually the case, then all the eigenvalues of N also have to be in the open right half plane. Conditions a) and b) are only sufficient if we allow conditionally stable systems, otherwise they are necessary and sufficient.

Corollary 4: Assume that each plant in $\mathscr{P}$ is open loop stable. Then there exists a single compensator C and a single $k^* > 0$ such that each plant in is stable with the control configuration shown in Fig. 1 for all $0 < k < k^*$ only if $\det(G(0))$ has the same sign for all plants in $\mathscr{P}$.

This result is disturbing because quite frequently systems are ill-conditioned and small uncertainties in the parameters can change the sign of the determinant. For example let

$$\begin{aligned} G_1(0) &= \begin{bmatrix} 1 & 1.05 \\ 1 & 1 \end{bmatrix}, \ \det G_1(0) = -0.05 \\ G_2(0) &= \begin{bmatrix} 1 & 1.05 \\ 0.95 & 1 \end{bmatrix}, \ \det G_2(0) = +0.0025 \end{aligned} \tag{11}$$

then independent of possible differences in the dynamics of the two systems it will be impossible to design any control scheme involving integral action which will be unconditionally stable for both G_1 and G_2.

On the other hand, it is often possible to find a compensator C such that all the eigenvalues of H(0) are in the RHP, though the eigenvalues of the different G(0) might not be restricted to a single half plane, as long as $\det G(0)$ does not change its sign. For example, let the family $\mathscr{P}$ be defined by

$$G(0) = \begin{bmatrix} \alpha & -1 \\ 1 & \alpha \end{bmatrix} \quad -0.5 \le \alpha \le +0.5 \tag{12}$$

The eigenvalues are $\lambda_{1,2} = \alpha \pm i$ and can lie in both half planes. Choose a constant compensator

$$C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{13}$$

to yield

$$H = G(0)G = \begin{bmatrix} 1 & \alpha \\ -\alpha & 1 \end{bmatrix} \tag{14}$$

with the eigenvalues $\lambda_{1,2} = 1 \pm i\alpha$ which lie always in the RHP.

It would almost not be worthwhile to state Theorem 4 because it is so similar to Theorem 1, were it not for a striking resemblance with a result obtained by Kwakernaak.*

Theorem 5 [4]: Suppose that the family of plants satisfies the following assumptions.

(a) Each plant in $\mathscr{P}$ is finite dimensional, is a nonsingular perturbation of the nominal plant such that $\Pi_\infty = \lim_{|s| \to \infty} \Pi(s)$ exists, has the same number of transmission zeros as the nominal plant, and is stabilizable and detectable.

(b) The transmission zeros of each plant all lie in a bounded region in the open left-half complex plane.

(c) There exists a fixed square matrix N with all its eigenvalues in the open right-half complex plant such that for each plant the matrix $\Pi_\infty N$ has all its eigenvalues in a bounded region in the open right-half complex plane.

Then there exists a single controller that stabilizes the control system for each plant in the family $\mathscr{P}$.

Let us analyze the similarities and differences between Theorem 4 and 5: Theorem 5 puts no restriction on the pole location of the different plants (the plants can be unstable) but requires the zeros to be in the LHP. Theorem 4 puts no restriction on the zero location of the different plants (the plants can be nonminimum phase) but requires the poles to be in the LHP.

Theorem 5 puts a restriction on the asymptotic behavior of the characteristic loci for $\omega \to \infty$; the CL of all the plants in $\mathscr{P}$ together with the compensator have to approach the origin from the same half plane. Theorem 4 puts a restriction on the asymptotic behavior of the CL as $\omega \to 0$; it is required that the CL of all the plants in $\mathscr{P}$ together with the compensator do not cross the negative real axis in the limit.

Let the plant be a single-input-single-output system

$$G(s) = k\chi(s)/\phi_p(s) \tag{15}$$

where ϕ_p is the plant characteristic polynomial, χ a monic polynomial and k a scalar constant. Then Theorem 5 requires that for each plant in $\mathscr{P}$ the constant k i.e. the gain at very high frequencies has the same sign. For single-input-single-output systems Theorem 4 requires the gain at very low frequencies (the steady state gain) of all the plants in $\mathscr{P}$ to have the same sign.

Thus Theorem 4 and 5 complement each other in an interesting manner and can be regarded as dual to each other.

Bifurcation Theory

All real systems are nonlinear and controllers are usually designed based on linearized models. Because of the system nonlinearities, the linearized model becomes invalid when the plant operating

* I am indebted to Prof. Kwakernaak for pointing out this resemblance.

conditions change and robustness problems may arise. We would like to investigate what type of system nonlinearities can cause robustness problems when integral control is employed.

For our discussion we will make use of some basic results of bifurcation theory. For details the reader is referred to the book by Iooss and Joseph [5]. Let the nonlinear system $\mathcal{N}$ be described by the equations

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \qquad \mathcal{N} \tag{16}$$

where $x \in R^m$, $y \in R^n$, $u \in R^n$ are the states, outputs and inputs respectively. For a particular u the steady state values of the variables - if they exist - are defined by the following equations

$$\begin{aligned} 0 &= f(x_s,u) \\ y_s &= h(x_s) \end{aligned} \qquad \mathcal{N}_s \tag{17}$$

Define $\tilde{A}(u) = \left.\frac{\partial f}{\partial x}\right|_{x_s,u}$; $\tilde{B}(u) = \left.\frac{\partial f}{\partial u}\right|_{x_s,u}$; $\tilde{C}(u) = \left.\frac{\partial h}{\partial x}\right|_{x_s,u}$

Result 1 (Static Bifurcation): If an eigenvalue of $\tilde{A}(u)$ vanishes for a particular input u, the system has multiple steady states. That is, depending on the initial conditions, the nonlinear system can settle to more than one steady state for the same constant input u. We will call this type of multiplicity output multiplicity because multiple outputs can result from the same input.

Result 2 (Hopf Bifurcation): If a pair of eigenvalues of $\tilde{A}(u)$ is purely imaginary for a particular input u, the system exhibits limit cycle behavior. That is, for a particular input u the system does not settle to a steady state but cycles continuously.

Result 3 (Implicit Function Theorem): The system of nonlinear algebraic equations $\mathcal{N}_s$ will have multiple solutions for some specified values of y_s if

$$\det \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & 0 \end{bmatrix} = -\det\tilde{A}\,\det(\tilde{C}\tilde{A}^{-1}\tilde{B}) = 0 \tag{18}$$

and therefore if

$$\det(\tilde{C}\tilde{A}^{-1}\tilde{B}) = \det(\tilde{G}(0)) = 0 \tag{18}$$

(Here we have made the assumption that $\det\tilde{A} \neq 0$ and we have applied Schur's determinant formula [6]). $\tilde{G}(0)$ is the steady state gain matrix of the linearized system. The existence of multiple solutions of $\mathcal{N}_s$ implies that there are several different steady state inputs u which bring the output y of the system $\mathcal{N}$ to the same steady state y_s. We call this type of multiplicity input multiplicity [7].

Let us now equip the system $\mathcal{N}$ with an integral controller to obtain the combined system

$$\begin{aligned} \dot{x} &= f(x,u) \\ \dot{z} &= y_d - y = y_d - h(x) \\ u &= kz \end{aligned} \qquad \mathcal{N}' \tag{19}$$

and after linearization

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \tilde{A} & k\tilde{B} \\ -\tilde{C} & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ y_d \end{bmatrix} \qquad \mathcal{L}' \tag{20}$$

Result 1 implies that the system $\mathcal{N}'$ will exhibit multiplicities if

$$\det \begin{bmatrix} \tilde{A} & k\tilde{B} \\ -\tilde{C} & 0 \end{bmatrix} = \det\tilde{A} \cdot \det(k\tilde{C}\tilde{A}^{-1}\tilde{B}) = 0 \tag{21}$$

and therefore if $\det(\tilde{C}\tilde{A}^{-1}\tilde{B}) = 0$ for some setpoint y_d. (Here we have made again the assumption that $\det A \neq 0$ and we have applied Schur's determinant formula [6]).

We have arrived at the following conclusions:

- Even though the open loop system might not have any static bifurcation point, the closed loop system with integral control can exhibit static bifurcation. This is the case if the open loop system has input multiplicities.
- If the open loop system has input multiplicities the determinant of the steady state gain matrix of the linearized system vanishes ($\det \tilde{G}(0) = 0$). Or, in other words, as u changes, the eigenvalues of the steady state gain matrix of the linearized system move through the origin from one half plane to the other. If the number of eigenvalues moving through the origin simultaneously is odd, $\det\tilde{G}(0)$ will switch signs. From Cor. 4 we know that under these circumstances there exists no single linear compensator C such that all the linearized plants in the family are stable with integral control or equivalently, such that the nonlinear plant is stable everywhere in its operating regime. Thus, when integral control is used, input multiplicities of nonlinear systems usually cause robustness problems which cannot be removed with linear compensators.

If the eigenvalues of the linearized system with integral control ($\mathcal{L}'$) are purely imaginary Hopf bifurcation occurs and the nonlinear system will exhibit limit cycle behavior (Result 2). From the preceding discussion on static bifurcation one would suspect that there is some connection between the occurrence of purely imaginary eigenvalues of $\mathcal{N}'$ and purely imaginary eigenvalues of $\tilde{G}(0)$. However, one can show through counterexamples that there is no correspondence between the two. That is, limit cycles of $\mathcal{N}'$ cannot be predicted from steady state information (i.e. $\mathcal{N}_s$) alone. No results on the existence of linear compensators to avoid limit cycle behavior when integral controllers are applied to nonlinear systems, are available to date.

Discussion

It could be argued that the definition of integral controllability (Def. 1) is too restrictive and that therefore all the other results of the paper are too conservative to be practically useful. In particular the maximum allowed gain k* for integral controllability might be very small and also there might be a conditionally stable region which is large enough to exclude robustness problems.

First of all, even for infinitesimally small controller gains k the closed loop system will display zero steady state tracking error though the dynamic response might be sluggish. Furthermore we will show next that for a large class of systems the lack of integral controllability implies also that these systems cannot be conditionally stable.

Theorem 6: There exists a k > 0 such that the closed loop system shown in Fig. 1 is stable only if $\det H(0) > 0$.

Proof: Let $H(s) = N(s)\, d(s)^{-1}$ where d(s) is the characteristic polynomial of H(s) and N(s) is a polynomial matrix. d(s) is monic and because of the stability assumption all its coefficients are positive. The closed loop system shown in Fig. 1 is stable only if all the coefficients of the closed loop characteristic polynomial $\det(sd(s)\,I+kN(s))$ are positive. The constant coefficient is $\det(kN(0))$ and therefore for closed loop stability it is required that $\det N(0) > 0$ and $\det H(0) > 0$.

A comparison of Thm. 6 with Thm. 1 and Cor. 1 shows that conditional stability without integral controllability is only possible if an even number of eigenvalues of $H(0)$ is in the open LHP. If the number of eigenvalues of $H(0)$ in the LHP is odd the closed loop system is unstable for all positive gains k. In particular if the steady state gain $H(0)$ of a single-input-single-output system is negative it is unstable for all positive gains.

Thm. 6 also strengthens considerably a result obtained previously [8] which states as a requirement for zero tracking error for step inputs $\det H(0) \neq 0$. We have shown here that unless $\det H(0) > 0$ the closed loop system is unstable.

Conclusion

A variety of results relating to the stability and robustness of linear and nonlinear systems with integral controllers has been derived. It is most significant that the conditions which have to be satisfied for the controller design to be feasible can all be obtained from steady state information about the plant. Several issues remain unresolved. Of particular importance is the question how many restrictions can be placed on the structure of the compensator which makes a system integral controllable and robust. Any restrictions imply a simplified control structure and are therefore practically significant.

Acknowledgement

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